12.2 Light Particles and Matter Waves

SECTION EXPECTATIONS

- Define and describe the concepts related to the understanding of matter waves.
- Describe how the development of quantum theory has led to technological advances such as the electron microscope.

KEY TERMS

- Compton effect
- de Broglie wavelength
- wave-particle duality

When Millikan's experimental results verified Einstein's interpretation of the photoelectric effect, the scientific community began to accept the particle nature of light. Physicists started to ask more questions about the extent to which particles of light, or photons, resembled particles of matter. U.S. physicist Arthur Compton (1892–1962) decided to study elastic collisions between photons and electrons. Would the law of conservation of momentum apply to such collisions? How could physicists determine the momentum (mv) of a particle that has no mass?

The Compton Effect

The ideal way to study collisions between particles is to start with free particles. Preferably, the only force acting on either particle at the moment of the collision is the impact of the other particle. However, electrons rarely exist free of atoms. So, Compton reasoned that if the photon's energy was significantly greater than the work function of the metal, the energy required to free an electron from the metal would be negligible when compared to the energy of the interaction. He needed a source of highly energetic photons.

About 30 years prior to Compton's work, Wilhelm Conrad Röntgen (1845–1923) discovered X rays and demonstrated that they are high-frequency electromagnetic waves. Thus, X-ray photons would have the amount of energy that Compton needed for his studies. In 1923, Compton carried out some very sophisticated experiments on collisions between X-ray photons and electrons. The phenomenon that he discovered is now known as the **Compton effect** and is illustrated in Figure 12.10. When a very high-energy X-ray photon collides with a "free" electron, it gives some of its energy to the electron and a lower-energy photon scatters off the electron.

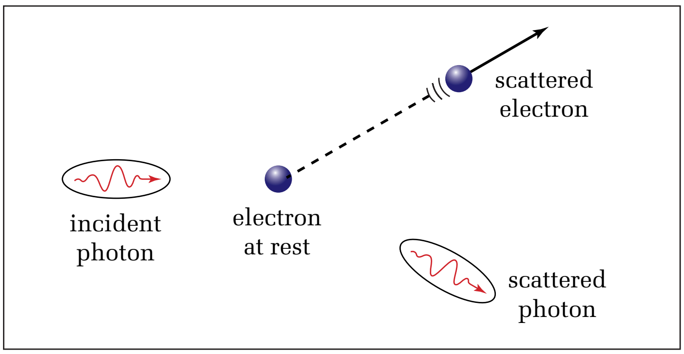

Figure 12.10 When a high-energy photon collides with a "free" electron, both energy and momentum are conserved.

You can describe mathematically the conservation of energy in a photon-electron collision as follows, where hf is the energy of the photon before the collision, hf' is the energy of the photon after the collision, and $\frac{1}{2}m_e v'^2$ is the kinetic energy of the electron after the collision.

$$hf = hf' + \frac{1}{2}m_e v'^2$$

Since the scattered photon has a lower energy, it must have a lower frequency and a longer wavelength than the original photon. Compton's measurements showed that the scattered photon had a lower frequency, and that kinetic energy gained by an electron in a collision with a photon was equal to the energy lost by the photon.

The more difficult task for Compton was finding a way to determine whether momentum had been conserved in the collision. The familiar expression for momentum, $p = mv$, contains the object's mass, but photons have no mass. So Compton turned to Einstein's now famous equation, $E = mc^2$, to find the mass equivalence of a photon. The following steps show how Compton used Einstein's relationship to derive an expression for the momentum of a photon. Since the goal is to find the magnitude of the momentum, vector notations are omitted.

- Write Einstein's equation that describes the energy equivalent of mass. $$E = mc^2$$
- Divide both sides of the equation by c^2 to solve for mass. $$m = \frac{E}{c^2}$$
- Write the equation for momentum. $$p = mv$$
- Substitute the energy equivalent of mass into the equation for momentum. $$p = \frac{E}{c^2}v$$
- Since the velocity of a photon is c, substitute c for v and simplify. $$P = \frac{E}{c^{\cancel{2}}}\cancel{c} = \frac{E}{c}$$
- Substitute the expression for the energy of a photon (hf) for E in the equation for momentum. $$p = \frac{hf}{c}$$
- The momentum of a photon is usually expressed in terms of wavelength, rather than frequency. Use the equation for the velocity of a wave to find the expression for f in terms of v. Note that the velocity of a light wave is c. $$f\lambda = v$$ $$f\lambda = c$$ $$f = \frac{c}{\lambda}$$
- Substitute the expression for frequency into the momentum equation and simplify. $$p = \frac{h\frac{\cancel{c}}{\lambda}}{\cancel{c}}$$ $$p = \frac{h}{\lambda}$$

When Compton calculated the momentum of a photon using $p = \frac{h}{\lambda}$, he was able to show that momentum is conserved in collisions between photons and electrons. These collisions obey all of the laws for collisions between two masses. The line between matter and energy was becoming more and more faint.

MOMENTUM OF A PHOTON

The momentum of a photon is the quotient of Planck's constant and the wavelength of the photon.

$$p = \frac{h}{\lambda}$$

Quantity	Symbol	SI unit
momentum	p	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$ (kilogram metres per seconds)
Planck's constant	h	$\text{J} \cdot \text{s}$ (joule seconds)
wavelength	λ	m (metres)

Unit Analysis

$$\frac{\text{kilogram} \cdot \text{metre}}{\text{second}} = \frac{\text{joule} \cdot \text{second}}{\text{metre}}$$

$$\frac{\text{kg} \cdot \text{m}}{\text{s}} = \frac{\text{J} \cdot \text{s}}{\text{m}} = \frac{\frac{\text{kg} \cdot \text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}}}{\cancel{\text{m}}} = \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

The following problem will help you to develop a feeling for the amount of momentum that is carried by photons.

SAMPLE PROBLEM

Momentum of a Photon

Calculate the momentum of a photon of light that has a frequency of 5.09×10^{14} Hz.

Conceptualize the Problem

- The *momentum* of a *photon* is related to its *wavelength*.
- A photon's *wavelength* is related to its *frequency* and the speed of light.

Identify the Goal

The momentum, p, of the photon

Identify the Variables and Constants

Known	Implied	Unknown
$f = 5.09 \times 10^{14}$ Hz	$c = 3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}$	λ
		p

Develop a Strategy

Find the wavelength by using the equation for the speed of waves and the value for the speed of light.

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}}{5.09 \times 10^{14}\ \text{s}^{-1}}$$

$$\lambda = 5.8939 \times 10^{-7}\ \text{m}$$

Use the equation that relates the momentum of a photon to its wavelength.

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{5.8939 \times 10^{-7}\ \mathrm{m}}$$

$$p = 1.1249 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

$$p \cong 1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

The momentum of a photon with a frequency of 5.09×10^{14} Hz is $1.12 \times 10^{-27}\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

Validate the Solution

You would expect the momentum of a photon to be exceedingly small, and it is. Check to see if the units cancel to give $\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$.

$$\frac{\mathrm{J \cdot s}}{\mathrm{m}} = \frac{\frac{\mathrm{kg \cdot m^{\cancel{2}}}}{\mathrm{s^{\cancel{2}}}} \cdot \cancel{\mathrm{s}}}{\cancel{\mathrm{m}}} = \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

PRACTICE PROBLEMS

1. Find the momentum of a photon with a wavelength of 1.55 m (radio wave).
2. Find the momentum of a gamma ray photon with a frequency of 4.27×10^{20} Hz.
3. What would be the wavelength of a photon that had the same momentum as a neutron travelling at 8.26×10^{7} m/s?
4. How many photons with a wavelength of 5.89×10^{-7} m would it take to equal the momentum of a 5.00 g Ping-Pong™ ball moving at 8.25 m/s?
5. What would be the frequency of a photon with a momentum of 2.45×10^{-32} kg · m/s? In what part of the electromagnetic spectrum would this photon be?

Matter Waves

By the 1920s, physicists had accepted the quantum theory of light and continued to refine the concepts. Once again, however, the scientific community was startled by the revolutionary theory proposed by a young French graduate student, who was studying at the Sorbonne. As part of his doctoral dissertation, Louis de Broglie (1892–1987) proposed that not only do light waves behave as particles, but also that particulate matter has wave properties.

De Broglie's professors at the Sorbonne thought that the concept was rather bizarre, so they sent the manuscript to Einstein and asked for his response to the proposal. Einstein read the dissertation with excitement and strongly supported de Broglie's proposal. De Broglie was promptly granted his Ph.D., and six years later, he was honoured with the Nobel Award in Physics for his theory of matter waves. The following steps lead to what is now called the **de Broglie wavelength** of matter waves.

- Write Compton's equation for the momentum of a photon. $p = \frac{h}{\lambda}$
- Solve the equation for wavelength. $\lambda = \frac{h}{p}$
- Substitute the value for the momentum of a particle for p. $\lambda = \frac{h}{mv}$

DE BROGLIE WAVELENGTH OF MATTER WAVES

The de Broglie wavelength of matter waves is the quotient of Planck's constant and the momentum of the mass.

$$\lambda = \frac{h}{mv}$$

Quantity	Symbol	SI unit
wavelength (of a matter wave)	λ	m (metres)
Planck's constant	h	J · s (joule seconds)
mass	m	kg (kilograms)
velocity	v	$\frac{\text{m}}{\text{s}}$ (metres per second)

Unit Analysis

$$\frac{\text{joule} \cdot \text{second}}{\text{kilogram}\frac{\text{metre}}{\text{second}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\text{J} \cdot \text{s}}{\text{kg}} \cdot \frac{\text{s}}{\text{m}} = \frac{\frac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}\cancel{\text{s}^2}}{\cancel{\text{kg}} \cdot \cancel{\text{m}}} = \text{m}$$

Note: Since wavelength is a scalar quantity, vector notations are not used for velocity.

SAMPLE PROBLEM

Matter Waves

Calculate the wavelength of an electron moving with a velocity of 6.39×10^6 m/s.

Conceptualize the Problem

- *Moving particles* have *wave* properties.
- The *wavelength* of particle waves depends on *Planck's constant* and the *momentum* of the particle.

Identify the Goal

The wavelength, λ, of the electron

Identify the Variables and Constants

Known	Implied	Unknown
$v = 6.39 \times 10^6 \ \frac{\text{m}}{\text{s}}$	$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$	λ
	$m_e = 9.11 \times 10^{-31}\ \text{kg}$	

Develop a Strategy

Use the equation for the de Broglie wavelength.

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\left(6.39 \times 10^6\ \frac{\text{m}}{\text{s}}\right)}$$

$$\lambda = 1.1389 \times 10^{-10}\ \text{m}$$

$$\lambda \cong 1.14 \times 10^{-10}\ \text{m}$$

The de Broglie wavelength of an electron travelling at 6.39×10^6 m/s is 1.14×10^{-10} m.

Validate the Solution

Since Planck's constant is in the numerator, you would expect that the value would be very small. Check the units to ensure that the final answer has the unit of metres.

$$\frac{\text{J} \cdot \text{s}}{\text{kg}\frac{\text{m}}{\text{s}}} = \frac{\frac{\cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}}{\text{s}^{\cancel{2}}} \cdot \cancel{\text{s}}}{\cancel{\text{kg}}} \cdot \frac{\cancel{\text{s}}}{\cancel{\text{m}}} = \text{m}$$

PRACTICE PROBLEMS

6. Calculate the wavelength of a proton that is moving at 3.79×10^6 m/s.
7. Calculate the wavelength of an alpha particle that is moving at 1.28×10^7 m/s.
8. What is the wavelength of a 5.00 g Ping-Pong™ ball moving at 12.7 m/s?
9. Find the wavelength of a jet airplane with a mass of 1.12×10^5 kg that is cruising at 891 km/h.
10. Calculate the wavelength of a beta particle (electron) that has an energy of 4.35×10^4 eV.
11. What is the speed of an electron that has a wavelength of 3.32×10^{-10} m?

To verify de Broglie's hypothesis that particles have wavelike properties, an experimenter would need to show that electrons exhibit interference. A technique such as Young's double-slit experiment would be ideal. This technique is not feasible for particles such as electrons, however, because the electrons have wavelengths in the range of 10^{-10} m. It simply is not possible to mechanically cut slits this small and close together. Fortunately, a new technique for observing interference of waves with very small wavelengths had recently been devised.

PHYSICS FILE

As you know, in 1897, J.J. Thomson provided solid evidence for the existence of the electron, a subatomic particle that is contained in all atoms. Ironically, just 30 years later, his son George P. Thomson demonstrated that electrons behave like waves.

During the 10 years prior to de Broglie's proposal, physicists Max von Laue (1879–1960) and Sir Lawrence Bragg (1890–1971) were developing the theory and technique for diffraction of X rays by crystals. The spacing between atoms in crystals is in the same order of magnitude as both the wavelength of X rays and electrons, about 10^{-10} m. As illustrated in Figure 12.11, when X rays scatter from the atoms in a crystal, they form diffraction patterns in much the same way that light forms diffraction patterns when it passes through a double slit or a diffraction grating. If electrons have wave properties, then the same crystals that diffract X rays should diffract electrons and create a pattern.

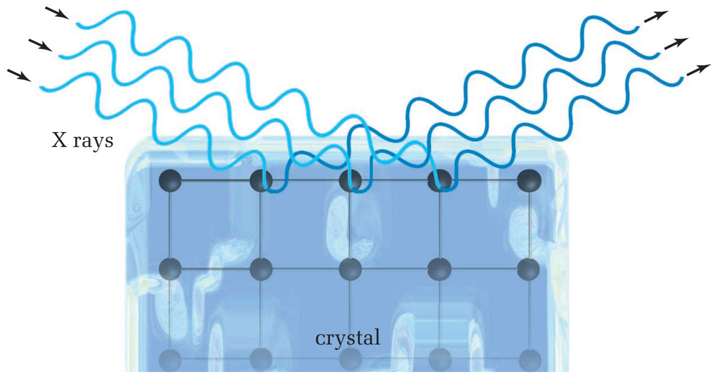

Figure 12.11 X rays scattered from regularly spaced atoms in a crystal will remain in phase only at certain scattering angles.

Within three years after de Broglie published his theory of matter waves, Clinton J. Davisson (1881–1958) and Lester H. Germer (1896–1971) of the United States and, working separately, George P. Thomson (1892–1975) of England carried out electron diffraction experiments. Both teams obtained patterns very similar to those formed by X rays. The wave nature of electrons was confirmed. In the years since, physicists have produced diffraction patterns with neutrons and other subatomic particles. Figure 12.12 shows diffraction patterns from aluminum foil formed by a beam of (A) X rays and (B) electrons.

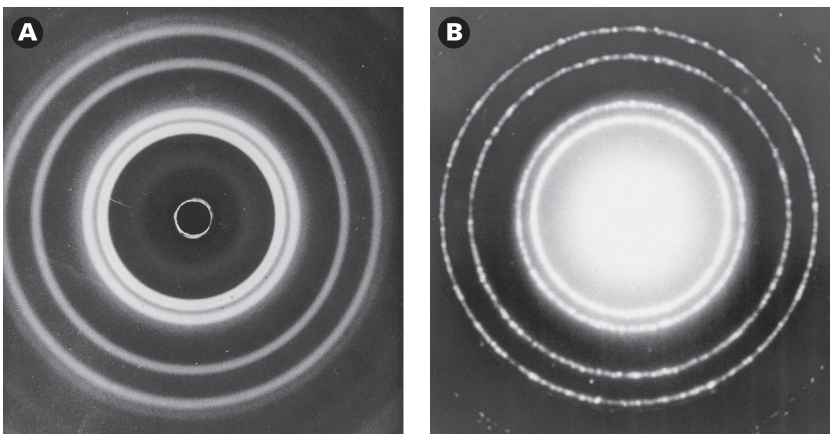

Figure 12.12 These patterns were created by diffraction of **(A)** X rays and **(B)** electrons by aluminum foil. Diffraction occurs as a result of the interference of waves. The similarity of these patterns verifies that electrons behave like waves.